II-6. Properties and Excitation of Spin Waves—A New Microwave Time Delay Medium

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Since the proposal of a method of coherent spin wave excitation by Schlömann[1] and its verification by Eshbach,[2] an interest in the use of coherent spin waves has arisen.

Spin waves are of interest for microwave time delay because of their short wavelengths and consequent low velocities. For example, for a 3000 Mc spin wave of wavelength $(5)(10^{-5})$ cm, the phase velocity is $(1.5)(10^{5})$ cm/sec. Assuming, for illustration, that the group velocity were equal to the phase velocity, then a pulse of spin waves launched on one side of a crystal of 1 cm length would arrive at the other side 6.7 microseconds later.

The pattern of electromagnetic propagation in such devices as microwave ferrimagnetic phase shifters is generally determined by simultaneous solution of Maxwell's equation and the equation of motion of magnetization, shown here in loss-free form:

$$\partial \vec{M}/\partial t = \gamma(\vec{M} \times \vec{H}). \tag{1}$$

Here $\vec{M}$ is the magnetization, $\vec{H}$ is the magnetic field intensity, and γ is the magnetogyric ratio.

The magnetic energy density corresponding to (1) is

$$f_m = \mu_o \vec{M} \cdot \vec{H} \quad \text{(mks)} \tag{2}$$

The wavelengths arrived at here are of the order of magnitude of the free-space wavelength.

While this type of propagation implies that the magnetization undergoes a phase shift, the phase change of the precession per spin separation is negligible. For example, for a lattice spacing of $(5)(10^{-8})$ cm and a wavelength of 10 cm, there are 10^8 spins between the two spins that precess with a phase difference of 180°. For such a minuscule rate of change of azimuthal precession angle per spin, Eq. (1) is quite satisfactory.

If a more rapid rate of change of precession phase exists, an additional energy term is added to the right side of (2). This "exchange" term depends on the rate of change of spin orientation and is given by:[3]

$$f_{ex} = A\left[(\nabla \alpha_1)^2 + (\nabla \alpha_2)^2 + (\nabla \alpha_3)^2\right] \quad \text{(cgs).} \tag{3}$$

Here α_i are direction cosines of magnetization with respect to a Cartesian coordinate system; and A is a constant of the material ($\sim 10^{-6}$ ergs/cm).

As a consequence, the magnetization is now governed by[4,5]

$$\frac{\partial \vec{M}}{\partial t} = \gamma(\vec{M} \times \vec{H}) + \gamma\left[\frac{2A}{|M_o|^2}\,\vec{M} \times \nabla^2 \vec{M}\right]. \tag{4}$$

For plane wave propagation, $\exp(j\omega t - j\vec{k} \cdot \vec{r})$, the secular equation corresponding to (4) is (5), which describes the well-known spin wave spectrum. Here ω_k is the spin wave frequency; H_i is the internal dc field; k is the wavenumber; θ is the angle between H_i and k.

$$\omega_k{}^2 = \gamma^2 \left[H_i + \frac{2A}{M_o} k^2 \right] \left[H_i + 4\pi M_o \sin\theta + \frac{2A}{M_o} k^2 \right] \quad \text{(cgs)}. \tag{5}$$

A more exact solution, in which Eq. (5) is solved simultaneously with Maxwell's equations, was arrived at by Auld,[6,7] who found the dispersion characteristics of Fig. 1 and gave expressions for $\vec{e}$ and $\vec{h}$ fields.

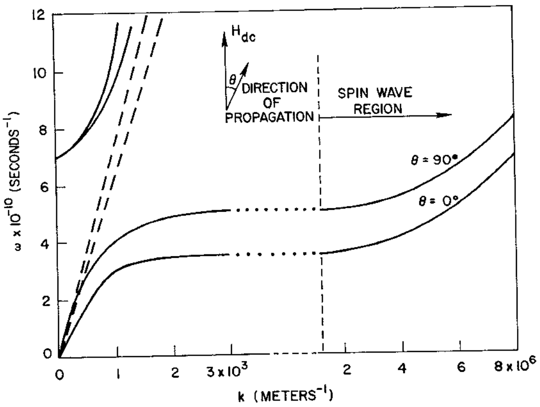

Fig. 1. Dispersion curves for uniform plane waves in an unbounded ferrite medium ($H_{dc} = 1750$ oe., $4\pi M_s = 1750$ oe.). The region of interest here is the spin wave region (after Auld[6]).

Because of the form of the expressions derived, the statement has been made that as the spin wave wavelength becomes shorter and shorter, the rf electric field becomes increasingly smaller compared with the rf magnetic field, and can eventually be neglected.[6,7]

We find that this is not the case, but that the converse is true. This is easily seen by substitution of a magnetic field intensity of form

$$(\hat{x}h_o + \hat{y}h_1) \exp[j(\omega t - kz]$$

into the Maxwell curl-H equation, in the absence of conduction current. The result is

$$\vec{e} = \left[\hat{x}h_1 \frac{k}{\omega \mathcal{E}_r \mathcal{E}_0} + \hat{y}(-h_o) \frac{k}{\omega \mathcal{E}_r \mathcal{E}_o} \right] \exp[j(\omega t - kz]; \tag{6}$$

so that

$$\left| \frac{e_x}{h_y} \right| = \left| \frac{e_y}{h_x} \right| = \frac{k}{\omega \mathcal{E}_r \mathcal{E}_o}. \tag{7}$$

As the wavelength λ_g gets shorter, k becomes larger, so that the ratio $|e/h|$ becomes greater. At 10 Gc, for $\mathcal{E}_r = 15$, and for $\lambda_g = (5)(10^{-7})$ meters, $|e/h| = (1.5)(10^6)$ ohms.

Direct substitution of the appropriate quantities into Auld's expressions yields essentially the same result.

Since spin waves are a magnetic, rather than an electric phenomenon, this is rather surprising. However, the mystery is cleared up by computing the magnetization. The results, summarized in Table I and Fig. 2, show that the rf magnetization m dominates the picture; the ratio m/e also varies directly as k.

TABLE I

Wave components for spin wave propagation. The dc magnetic field is in the z-direction. Propagation is in the Y-Z Plane. (Note that $\vec{m}$ is circularly polarized in both cases; $\vec{e}$, $\vec{h}$ and $\vec{b}$ are circularly polarized only for $\theta = 0^\circ$).

	$\theta = 0^\circ$	$\theta = 90^\circ$
$\vec{m}$	$m_o \begin{bmatrix}-1\\ j\\ 0\end{bmatrix} e^{j(\omega t - kz)}$	$m_o \begin{bmatrix}1\\ -j\\ 0\end{bmatrix} e^{j(\omega t - ky)}$
$\vec{e}$	$-m_o\left(\frac{k_f}{k}\right)\left(\frac{\mu_o}{\varepsilon}\right)^{1/2}\begin{bmatrix}-j\\ 1\\ 0\end{bmatrix} e^{j(\omega t - kz)}$	$-m_o\left(\frac{k_f}{k}\right)\left(\frac{\mu_o}{\varepsilon}\right)^{1/2}\begin{bmatrix}0\\ 0\\ -1\end{bmatrix} e^{j(\omega t - ky)}$
$\vec{h}$	$m_o\left(\frac{k_f}{k}\right)^2\begin{bmatrix}1\\ j\\ 0\end{bmatrix} e^{j(\omega t - kz)}$	$m_o \begin{bmatrix}(k_f/k)^2\\ j\\ 0\end{bmatrix} e^{j(\omega t - ky)}$
$\vec{b}$	$\simeq \mu_o m_o \begin{bmatrix}-1\\ j\\ 0\end{bmatrix} e^{j(\omega t - kz)}$	$\mu_o m_o \begin{bmatrix}1\\ 0\\ 0\end{bmatrix} e^{j(\omega t - ky)}$

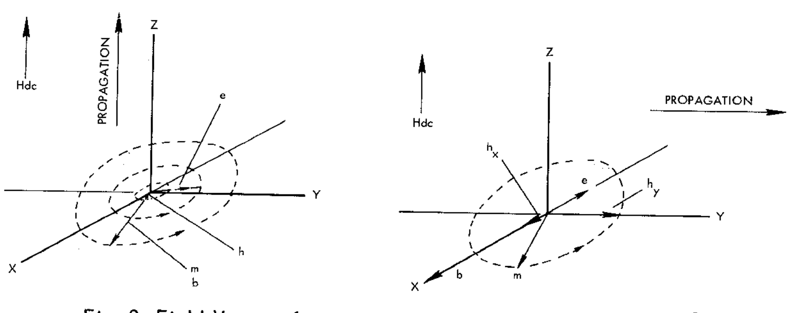

Fig. 2. Field Vectors for spin wave propagation. Left: for $\theta = 0^\circ$. Right: for $\theta = 90^\circ$.

It is of interest to compute the field intensities at a particular level of power. These are given in Table II, for spin wave propagation in a typical case ($4\pi M_o = 2439$ gauss; $A = (4.9)(10^{-7})$ erg/cm; $\varepsilon_r = 14$; precession angle 1°; 3000 Mc). Here p_{em} is the electromagnetic power density, computed as $\vec{E} \times \vec{H}$; p_{mag} is the magnetization energy power flow, computed by multiplication of the stored energy ($f_m + f_{ex}$) by the group velocity.[8]

TABLE II
Typical amplitudes for spin wave propagation.
$p_{em} = 6.0 \times 10^{-9}$ watts/cm^2 $p_{mag} = 3.5 \times 10^{-2}$ watts/cm^2 $E_x = 12.8$ volts per meter $H_x = 4.7 \times 10^{-6}$ amperes per meter $M_x = 3.4 \times 10^{3}$ amperes per meter $B_x = 4.3 \times 10^{-3}$ web. per meter^2

The reason for considering spin waves for microwave time delay lies in their slow propagation velocities. For z-directed spin waves, the dispersion relation is $\omega_k = |\gamma| \left[H_i + \frac{2A}{M_o} k^2\right]$. The phase velocity is

$$v_p = \omega_k \,[M_o/2A)\,(\omega_k\,|\gamma| - H_o)]^{1/2};$$

the group velocity is

$$v_g = 2\,|\gamma|\,(2A/M_o)^{1/2}\,(\omega_k\,|\gamma| - H_o)^{1/2}.$$

Both are variable with H_o. As in a waveguide at cutoff, $v_p = \infty$ when $v_g = 0$. Extrema of these functions are given in Tables III and IV.

TABLE III

Extrema in velocities, "guide" wavelengths and delay rates for spin waves.

Velocity	Minimum Value	Maximum Value	Occurs at $H_o =$
v_p	$\left(\frac{2A}{M_o}\,|\gamma|\omega_k\right)^{1/2}$		0
		∞	$\omega_k/|\gamma|$
v_g	0		$\omega_k/|\gamma|$
		$2\left(\frac{2A}{M_o}\,|\gamma|\omega_k\right)^{1/2}$	0

TABLE IV

Values of the Extrema for $f = 3000$ Mc, $4\pi M_o = 2439$ gauss; $A = 4.9 \times 10^{-7}$ erg/cm.

	At $H_o = 0$	At $H_o = \omega_k/|\gamma|$
v_p	$(4.1)(10^4)$ cm/sec	∞
λ_g	$(1.4)(10^{-5})$ cm	∞
v_g	$(8.2)(10^5)$ cm/sec	0
Delay Rate	12 microseconds/cm	∞

Although Table III shows that very large delay rates are possible, high losses will probably not allow them in practice. We are particularly interested in delay rates up to 100 microseconds/cm, where $\lambda_g \simeq 10^{-5}$ to 10^{-4} cm. The crucial problem is how to excite spin waves of such wavelengths.

If we consider excitation by a circularly polarized field intensity

$$(\hat{x}h_o + \hat{y}jh_o)\, \varepsilon^{j\omega t},$$

with

$$\vec{H}_{dc} = \hat{z}H_o,$$

we find, for z-directed spin waves, that

$$\frac{\partial^2 m_o}{\partial z^2} + k^2 m_o = -\frac{M_o^2}{2A} h_o. \tag{8}$$

Here all rf quantities vary as $\exp(j\omega t)$; m_o is $(m_x + jm_y)$. Since Eq. (8) is recognized as the lossless transmission line equation with excitation, it is convenient to examine it in terms of a transmission line analog.

We consider, for example, the analog parallel wire transmission line of Fig. 3, which has been excited by a shunt current at location ξ. By the usual transmission line analysis, the current at z is found to be

$$I(z) = I\left(\cos\frac{2\pi\xi}{\lambda}\right)\varepsilon^{j(\omega t - 2\pi z/\lambda)}, \tag{9}$$

where λ is the wavelength.

If I is replaced by linear incremental current density, $J(\xi)$, distributed from $\xi = 0$ to $\xi = a$, we get

$$I(z) = \varepsilon^{j\omega t}\int_{\xi=0}^{a} J(\xi)\, d\xi\; \varepsilon^{-j(2\pi z/\lambda)} \cos\frac{2\pi\xi}{\lambda}. \tag{10}$$

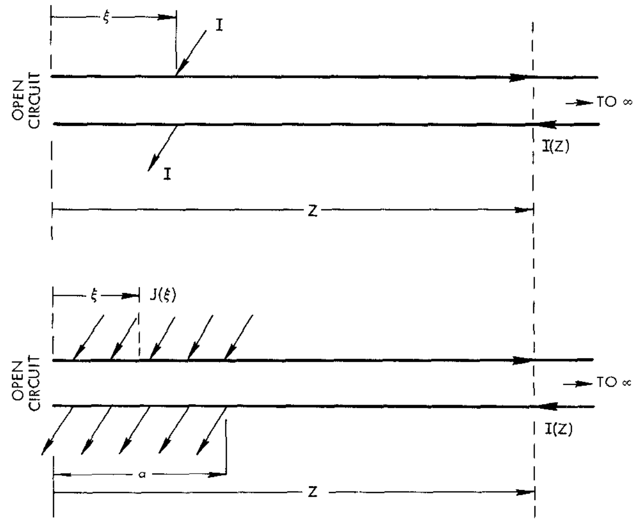

Fig. 3. Parallel wire transmission line excited by shunt current. Top: excitation at one point. Bottom: distributed excitation.

If $J(\xi)$ has a constant and in-phase value, J_c, from 0 to a, we find that $I(z) = (\lambda/2\pi)(J_c)$ for $a = \lambda/4$, $3\lambda/4$, ... Everywhere else it is less, falling to zero if $a = \lambda/2$, λ, ... In general, even though J_c extends over longer and longer distances, $I(z)$ does not increase above $(\lambda J_c/2\pi)$. Consequently, only the first $\lambda/4$ is effective; the energy fed into the line cannot be increased by merely increasing the length over which J_c extends.

This situation is the picture in spin wave excitation. Normal microwave fields extend, with essentially equal amplitude and phase, over distances of about 10^{-1} cm; however, in the above we considered wavelengths of 10^{-5} to 10^{-4} cm. As a result, we would expect to couple little energy from electromagnetic waves to spin waves.

One scheme for increasing the coupling,[1,2] makes use of the change in wavelength of spin waves effected by a change in H_o. Its transmission line analog is a propagating hollow metallic waveguide, which contains a section that operates very near cutoff and has a long guide wavelength. This technique causes the system to have a net dipole moment, increasing the coupling length to several times λ_g.

Another coupling scheme, useful for generating cylindrical spin waves, is proposed. Instead of immersing a ferrimagnetic material into a uniform field, we propose to excite spin waves by an intense, highly localized field. Such a field exists in the vicinity of a very fine wire, such as the 34-microinch Wollaston wire of a microwave bolometer. For example, for an absorbed power (300°K) in such a wire (length 0.2 cm) of 10^{-3} watts, the magnetic field intensity at the surface is 11 oersteds, dropping to 5.5 oersteds in $(4.3)(10^{-5})$ cm. A diagram of the proposed scheme is shown in Fig. 4. An estimate of the degree of coupling possible by such a scheme has been made. Under the assumption that the efficiency with which energy will actually be coupled into cylindrical spin waves is approximately that which would be coupled to a ferrimagnetic slab of height equal to the diameter of the exciting wire (Fig. 5), the conversion efficiency of this transducer in a nonresonant coupling system is predicted to be about 5%.

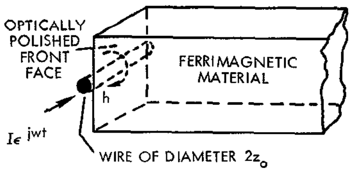

Fig. 4. Spin wave excitation by fine wire.

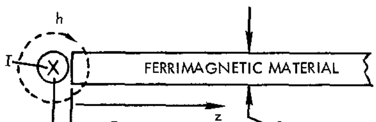

Fig. 5. Model for estimating coupling in arrangement of Fig. 4.

We are presently investigating the general coupling problem by such microscopic exciting systems by a Fourier integral technique. Progress on this aspect of the problem will be reported in the presentation.

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4. In this paper, the subject of magnetoelastic coupling has been omitted. While such coupling will influence spin wave propagation, it will not alter the principles discussed in this paper.
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